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A NON-NEWTONIAN TWO-PHASE FLUID MODEL for BLOOD FLOW THROUGH ARTERIES UNDER STENOTIC CONDITION

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ABSTRACT

This article is concerned with the theoretical analysis of the problem of overlapping stenosis on blood flow in arterial segment. A macroscopic two phase model of the blood has been utilized in this study to account for the presence of red cells (erythrocytes) in plasma. The coupled differential equations governing the flow of fluid (plasma) and the particle phases are solved by a combined use of analytical and numerical techniques with appropriate boundary conditions. The geometry of the asymmetric shape of the stenosis assumed to be manifested in the arterial segment is given due consideration in the analysis. An extensive quantitative analysis has been performed for narrowing of vessels through numerical computations on flow resistance, shear stress at stenosis throat and at the critical height of the stenosis. The increasing behaviour of the flow resistance is observed in the analysis when the stenosis height as well as the hematocrit increases. It is further revealed that the shear stress at stenosis throat and at critical height increases for increasing value of stenosis height in the permissible range of hematocrit. The results are compared with the available data presented by previous researchers.

KEYWORDS

Stenosis, Hematocrit, Impedance and Shear stress.

INTRODUCTION

The cause and development of many arterial diseases leading to serious circulatory disorder depends much on the flow characteristics of the streaming blood together with the mechanical behaviour of the blood vessel walls. Stenosis – a generic medical term means narrowing of an artery, tube or orifice caused by intravascular plaques in the arterial system of the human body. As a result unusual growths are developed at various locations of the cardiovascular system leading to reduction of normal pattern of blood flow through the constricted region of an artery. The presence of stenosis in major blood vessels that supply blood to the brain, heart and other organs may lead to stroke, heart attack and many other cardiovascular diseases. Cardial ischemia is caused due to the constriction responsible for insufficient flow of blood through the coronary arteries into the heart. High grade stenosis increases flow resistance in arteries. In order to maintain the necessary blood supply, the body is forced to raise the blood pressure. The high blood pressure and the arterial constriction together increase flow velocity, shear stress and decrease pressure substantially at the stenosis throat leading to thrombus formation. If the disease takes a severe form it may lead to serious circulatory disorders, morbidity or even fatality.

Although the exact mechanism of the formation of stenosis in the arterial lumen is not understood from the standpoint of physiology/pathology, it is established that the rheologic and fluid dynamic properties of blood

and blood flow could play a significant role in the basic understanding, diagnosis and treatment of this arterial disease. In the recent past quite a good number of analytical as well as experimental studies have been investigated by several researchers to explore the effect of arterial constriction on the flow characteristics of blood. Some of these studies were performed on the assumption that the fluid representing blood is Newtonian with stenotic geometry represented by a smooth cosine function.

Experimental observations reveal that blood, being predominantly a suspension of erythrocytes in plasma behaves like a non-Newtonian fluid at low shear rates [4, 5]. During its flow through micro vessels, especially in diseased states clotting affects small arteries. A number of theoretical studies [7] and experimental observations[6] suggest that blood cannot be treated as a single phase homogeneous viscous fluid in narrow arteries of diameter ≤ 1000μm. A more comprehensive study on the diagnosis, prevention and treatment of stenosis related diseases suggest that an accurate description of blood flow requires consideration of erythrocytes (red cells) as discrete particles in small arteries [9]. In view of their observations it is preferable to represent the flow of blood in narrow tubes by a two layered model instead of one layered model. In a recent paper Srivastava [7] presented a brief survey of the literature on two phase blood flow in the stenosed artery. Most of the research works were focused on a single stenosis (symmetric or non-symmetric) model. On the basis of experimental observations it is established that stenosis may develop in series or may also overlap.

In view of the above, a mathematical model is developed to study the effects of an overlapping stenosis on flow behaviour of the streaming blood to be treated as macroscopic two phase fluid (i.e. suspension of erythrocytes in plasma). The geometry of the overlapping stenosis to be manifested in the arterial segment is given due consideration in the present analysis. The most important rheological parameter involved in the investigation is hematocrit. It is essentially the percentage of blood made up of red blood cells (R.B.C.'s). As red blood cells are the portion of the blood that carry oxygen, so hematocrit is a representation of the oxygen carrying capacity of the blood. Its normal values for adults – males: 40-54% and females: 36-46%.

An extensive quantitative analysis is carried out by performing large scale numerical computations of the measurable flow variables having more physiological significance by developing computer codes. Their graphical representations are presented at the end of the paper with appropriate scientific discussions. Finally comparisons are made with the other existing results to justify the applicability of the present model.

MATERIALS AND METHODS:

Let us consider the axi-symmetric flow of blood through an artery with an overlapping stenosis. The geometry of the stenosis [3] assumed to be manifested in the arterial segment given by is described in

Fig.1 as:

$$\frac{R(z)}{R_0} = 1 - \frac{3}{2}\frac{\delta}{R_0 L_0^4}[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4], \quad d \le z \le d + L_0$$

$$= 1, \quad \text{otherwise} \qquad (1)$$

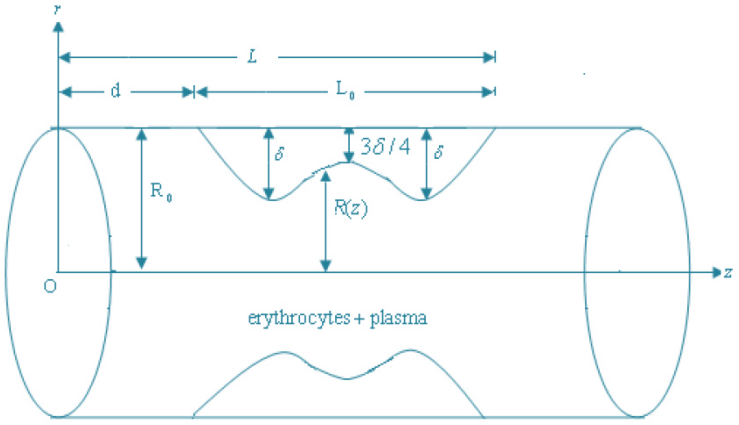

Figure 1 Geometry of an overlapping stenosis

where $R(z)$ and R_0 are the radius of the tube with and without stenosis, L_0 is the length of the stenosis, d indicates its location and δ is the maximum height of the stenosis. $\frac{3\delta}{4}$ is the height of the stenosis at z = d + $\frac{L_0}{2}$ which is called the critical height.

Here blood is assumed to be represented by a non-Newtonian two-phase macroscopic model.

The equations [11] describing the steady flow of a two-phase macroscopic model of blood may be expressed as:

$$(1-C)\rho_f\{u_f\frac{\partial u_f}{\partial z}+v_f\frac{\partial u_f}{\partial r}\}=-(1-C)\frac{\partial p}{\partial z}+(1-C)\mu_s(C)\nabla^2 u_f+CS(u_p-u_f) \quad (2)$$

$$(1-C)\rho_f\{u_f\frac{\partial v_f}{\partial z}+v_f\frac{\partial v_f}{\partial r}\}=-(1-C)\frac{\partial p}{\partial r}+(1-C)\mu_s(C)(\nabla^2-\frac{1}{r^2})v_f+CS(v_p-v_f) \quad (3)$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r(1-C)v_f]+\frac{\partial}{\partial z}[(1-C)u_f]=0 \quad (4)$$

$$C\rho_p\{u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\}=-C\frac{\partial p}{\partial z}+CS(u_f-u_p) \quad (5)$$

$$C\rho_p\{u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\}=-C\frac{\partial p}{\partial r}+CS(v_f-v_p) \quad (6)$$

$$\frac{1}{r}\frac{\partial}{\partial r}[rCv_p]+\frac{\partial}{\partial z}[Cu_p]=0 \quad (7)$$

Where $\nabla^2=(\frac{1}{r})\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})+\frac{\partial^2}{\partial z^2}$ is the Laplacian operator, r and z are the cylindrical polar coordinate system where z measured along the axis of the tube and r perpendicular to the axis of the tube. Here u_f and v_f are the axial and radial components of the fluid velocities while u_p and v_p are the axial and radial components of the particle velocities. ρ_f and ρ_p are the actual density of the portion constituting the fluid(plasma) and the particle(erythrocyte) phases respectively, (1-C) ρ_f is the fluid

phase and C ρ_p is the particle phase densities where C stands for the volume fraction density of the particles which is known as Hematocrit, p is the pressure, $\mu_s(C)$; μ_s is the effective viscosity, S is the drag coefficient for the force exerted by one phase on another. The Hematocrit C is chosen to be constant. From the reports of Young [17], Srivastava and Rastogi[13], the governing equations of the laminar, steady, one-dimensional flow of blood in an artery, with a mild stenosis are derived [14] from equations (2) – (7) as

$$(1-C)\frac{dp}{dz} = (1-C)\frac{\mu_s(C)}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})u_f + Cs(u_p - u_f) \quad (8)$$

$$C\frac{dp}{dz} = Cs(u_f - u_p) \quad (9)$$

The expressions for the drag coefficient of interaction(S) and the effective viscosity (μ_s) are taken from (Srivastava & Srivastava) [14] as

$$S = 4.5(\frac{\mu_0}{a_0^2})\frac{\{4+3(8C-3C^2)^{\frac{1}{2}}+3C\}}{(2-3C)^2} \quad (10)$$

$$\mu_s(C) \approx \mu_s = \frac{\mu_0}{1-mC}$$
$$m = 0.07\exp[2.49C + (1107/T)\exp(-1.69C)] \quad (11)$$

T is measured in absolute temperature (K), μ_0 is the constant plasma viscosity and a_0 is the radius of a red cell.

The boundary conditions are

u_f =0 at r=R (z) (12)

$$\frac{\partial u_p}{\partial r} = 0 \quad \text{at} \quad r=0 \quad (13)$$

Integrating equations (8) and (9) and using the boundary conditions (12) and (13) we obtain the expressions for the velocity of fluid and particle phases as

$$u_f = -\frac{R_0^4}{4(1-C)\mu_s}\frac{dp}{dz}\{(\frac{R}{R_0})^2 - (\frac{r}{R_0})^2\} \quad (14)$$

$$u_p = -\frac{R_0^4}{4(1-C)\mu_s}\frac{dp}{dz}\{(\frac{R}{R_0})^2 - (\frac{r}{R_0})^2 + \frac{4(1-C)\mu_s}{SR_0^2}\} \quad (15)$$

The flow flux, Q is calculated as

$$Q = 2\pi(1-C)\int_0^R ru_f dr + 2\pi C\int_0^R ru_p dr$$
$$= -\frac{\pi R_0^4}{8(1-C)\mu_s}\frac{dp}{dz}\{(\frac{R}{R_0})^4 + \eta(\frac{R}{R_0})^2\} \quad (16)$$

Where $\eta = \frac{8C(1-C)\mu_s}{SR_0^2}$, a non-dimensional parameter.

From equation (16) we obtain the pressure gradient as

$$\frac{dp}{dz} = -\frac{8(1-C)\mu_s}{\pi R_0^4}\{\frac{1}{(\frac{R}{R_0})^4 + \eta(\frac{R}{R_0})^2}\} \quad (17)$$

The pressure drop, Δp across the stenosis in a tube of length L is obtained from equation (17) as

$$\Delta p = \int_0^L (-\frac{dp}{dz})dz = \frac{8(1-C)\mu_s Q}{\pi R_0^4}\xi, \quad (18)$$

where

$$\xi = \int_0^L \phi(z)dz = \int_0^d [\phi(z)]_{\frac{R}{R_0}=1} dz + \int_d^{d+L_0} [\phi(z)]dz + \int_{d+L_0}^L [\phi(z)]_{\frac{R}{R_0}=1} dz \quad (19)$$

We use numerical integration to evaluate ξ. We now obtain the expressions for impedance λ, the shear stress at stenosis throats, τ_s and the shear stress at the critical height of the stenosis, τ_c in their non-dimensional form as

$$\lambda = (1-C)\mu\{\frac{1-\frac{L_0}{L}}{1+\eta} + \frac{1}{L}\int_d^{d+L_0} \frac{dz}{(\frac{R}{R_0})^4 + \eta(\frac{R}{R_0})^2}\} \quad (20)$$

$$\tau_s = \frac{(1-C)\mu}{(1-1.25\frac{\delta}{R_0})^3 + \eta(1-1.25\frac{\delta}{R_0})} \quad (21)$$

$$\tau_c = \frac{(1-C)\mu}{\{1-0.75\frac{\delta}{R_0}\}^3 + \eta(1-0.25\frac{\delta}{R_0})} \quad (22)$$

where $\lambda = \frac{\overline{\lambda}}{\lambda_0}$, $\tau_s = \frac{\overline{\tau_s}}{\tau_0}$, $\tau_c = \frac{\overline{\tau_c}}{\tau_0}$,

$\overline{\lambda} = \frac{\Delta p}{Q}$, $\overline{\tau_s} = [-\frac{R}{2}(\frac{dp}{dz})]_{\frac{R}{R_0}=(1-1.25\frac{\delta}{R_0})}$, $\overline{\tau_c} = [-(\frac{R}{2})\frac{dp}{dz}]_{\frac{R}{R_0}=(1-0.75\frac{\delta}{R_0})}$, $\mu = \frac{\mu_s}{\mu_0}$,

$\lambda_0 = \frac{8\mu_0 L}{\pi R_0^4}$, $\tau_0 = \frac{4\mu_0 Q}{\pi R_0^3}$,

λ_0 and τ_0 are the flow resistance and shear stress for a normal artery(without stenosis) in Newtonian fluid i.e. in the absence of Hematocrit(C=0).

RESULTS AND DISCUSSIONS:

A specific numerical illustration is presented in the present analysis considering some particular values of the different physical and rheological parameters. The purpose of this numerical computation is to examine the validity of the model under consideration. For the present computational study, the following values of the parameters are taken [13,15]: d(cm)= 0.25,0.5 ; L_0 (cm)= 0.5, 0.75 ; L(cm) = 1; C = 0, 0.2, 0.4, 0.6; $\frac{\delta}{R_0}$ (non-dimensional stenosis height) = 0, 0.05,0.10,0.15. It is to note that C=0 corresponds

to the case of a Newtonian fluid and $\frac{\delta}{R_0}$ = 0 represents the absence of any stenosis.

An attempt is made through Figure 2 and Figure 3 to show the variation of flow resistance with stenotic height for values of Hematocrit(C) in the admissible range. As the stenotic height increases the numerical value of λ increases. In both the figures λ increases with increase in C which is consistent with the observations of Medhavi[19]. λ is almost constant for change of values of $\frac{\delta}{R_0}$ for any specific C. The stenotic height ($\frac{\delta}{R_0}$) is restricted upto 0.15 as beyond this value a separation of flow might occur even at a relatively small Reynolds's number [15]

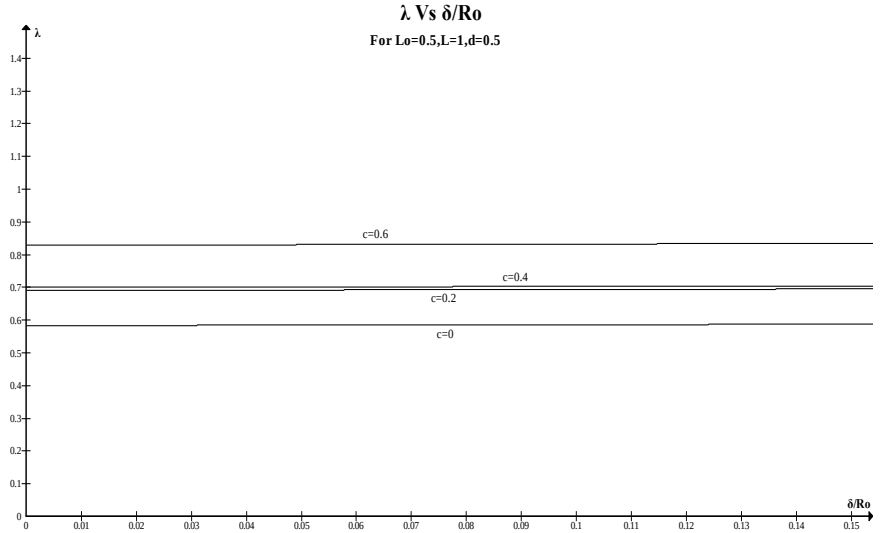

Figure 2 Variation of impedance, λ with $\frac{\delta}{R_0}$ for different C

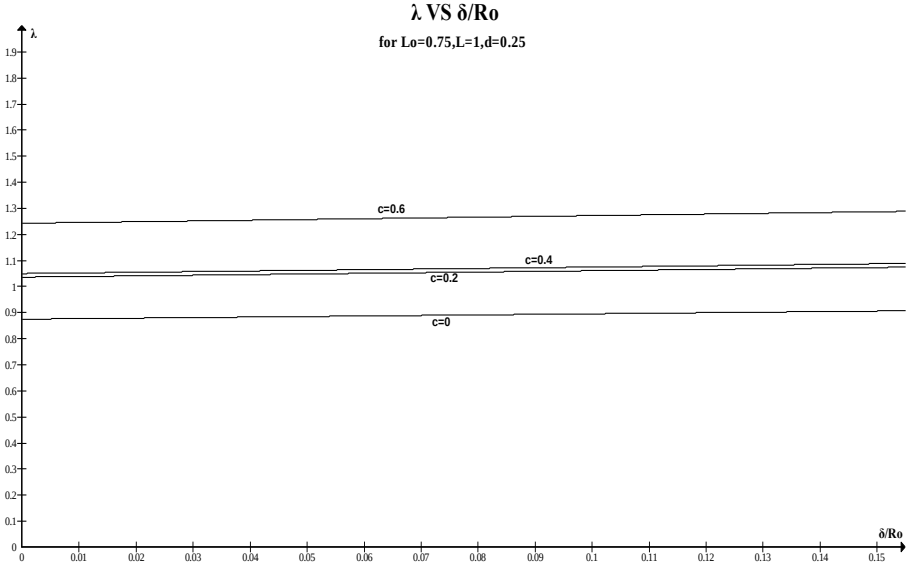

Figure 3 Variation of impedance, λ with $\frac{\delta}{R_0}$ for different C

Figure 4 depicts the variation of impedance with respect to Hematocrit C. It is observed that the increase in the value of the stenotic height does not influence substantially the flow resistance λ

if the stenotic region is 50% i.e. if L_0=0.5 and d = 0.5. But if d = 0.25 and L_0 = 0.75 then the flow resistance increase for the increase in C and $\frac{\delta}{R_0}$ (**Figure 4**).The graph shows asymptotic nature beyond C = 0.5 for $\frac{\delta}{R_0}$ = 0, 0.05,0.10,0.15. The findings are quite close to results of Medhavi[19].

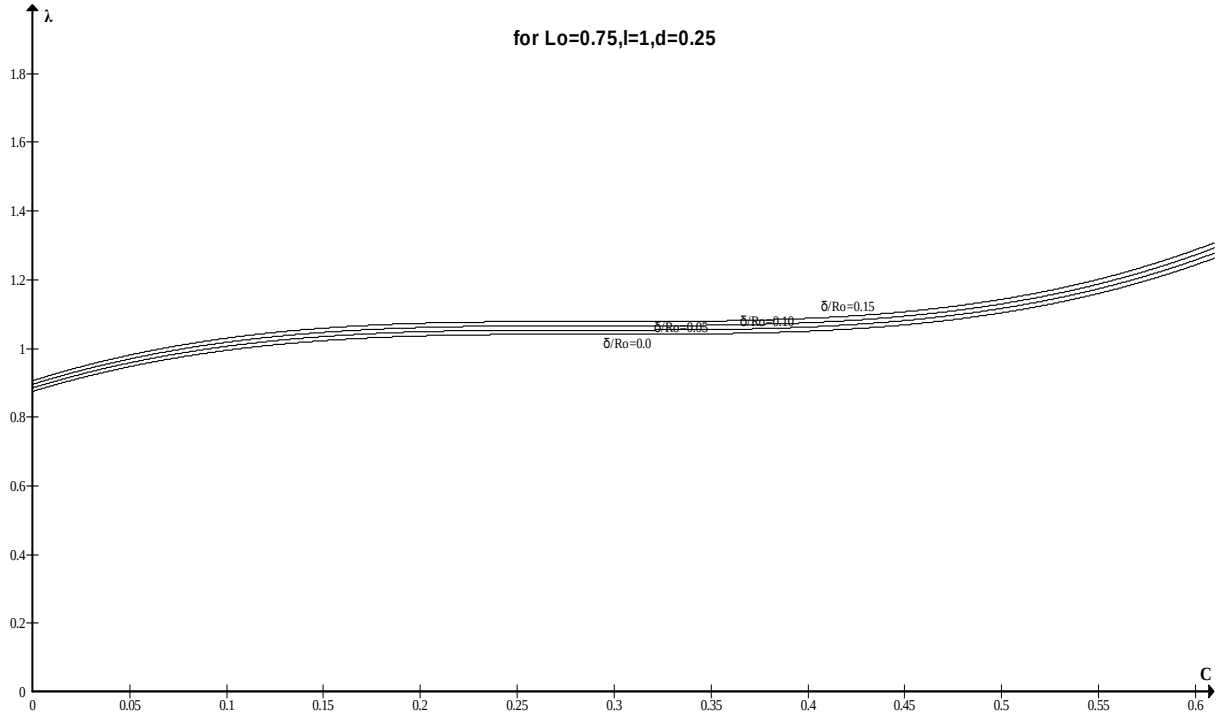

Figure 4 Variation of impedance, λ with C for different $\frac{\delta}{R_0}$

For different values of $\frac{\delta}{R_0}$, the pattern of the shear stress at stenosis throats, τ_s is shown in **Figure 5**. It is seen that τ_s increases with $\frac{\delta}{R_0}$ for C = 0, 0.2, 0.4, 0.6. Also for increasing value of C and for a specific value of $\frac{\delta}{R_0}$, the shear stress shows increasing behavior (**Figure 6**). C = 0.4 is an ideal case for both male and female and the result almost coincides for C = 0.2 and C = 0.4.

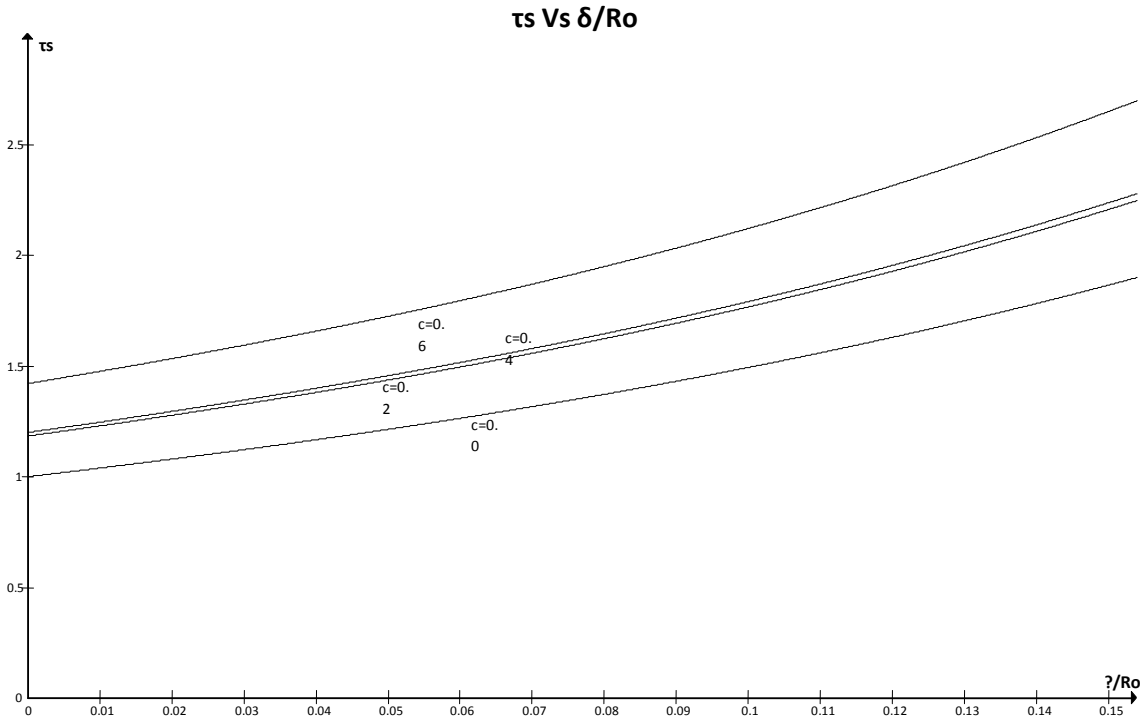

Figure 5 Variation of shear stress at stenosis throats, τ_s with $\frac{\delta}{R_0}$ for different C

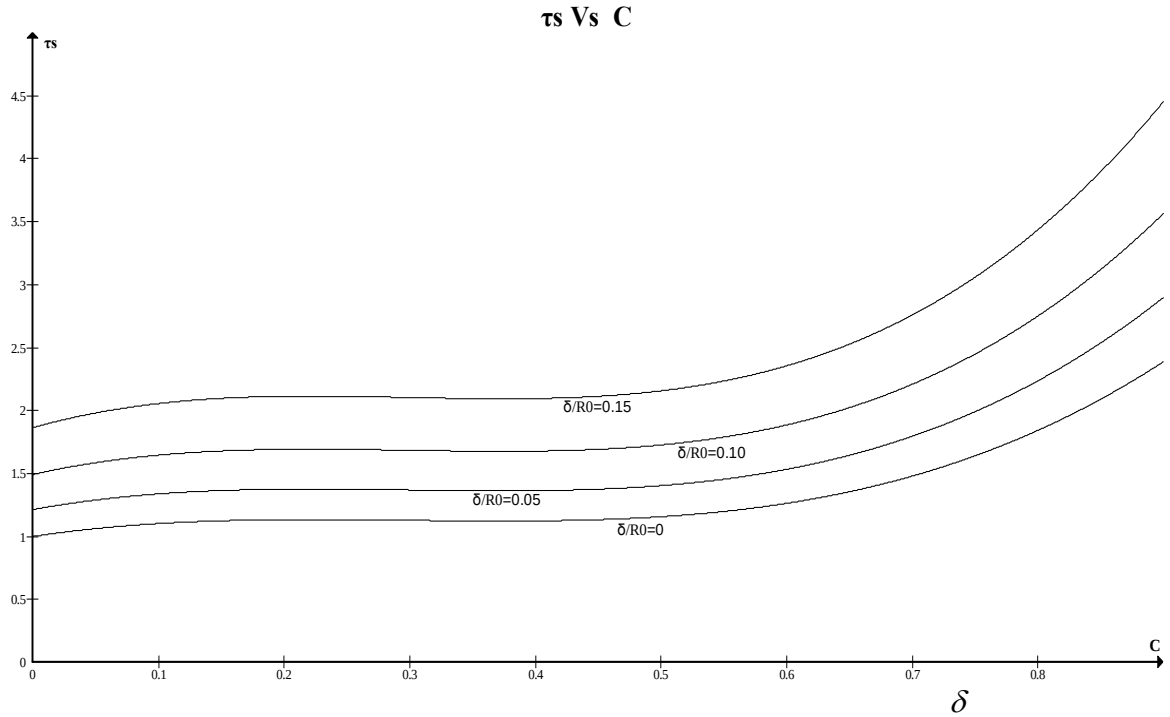

Figure 6 Variation of shear stress at stenosis throats, τ_s with C for different $\frac{\delta}{R_0}$

The magnitude of shear stress at stenosis throats τ_s is considerably higher than the flow resistance for the same values of C and $\frac{\delta}{R_0}$ (**Figure 2, 3 and 6**). Also τ_s shows asymptotic nature beyond the value 0.6 for C (**Figure 6**) which is consistent with the definition of C i.e. C normally lies between 0.2 and 0.55.

The shear stress at critical stenosis height, τ_c increases with $\frac{\delta}{R_0}$ for different values of C = 0, 0.2, 0.4, 0.6 (**Figure 7**)

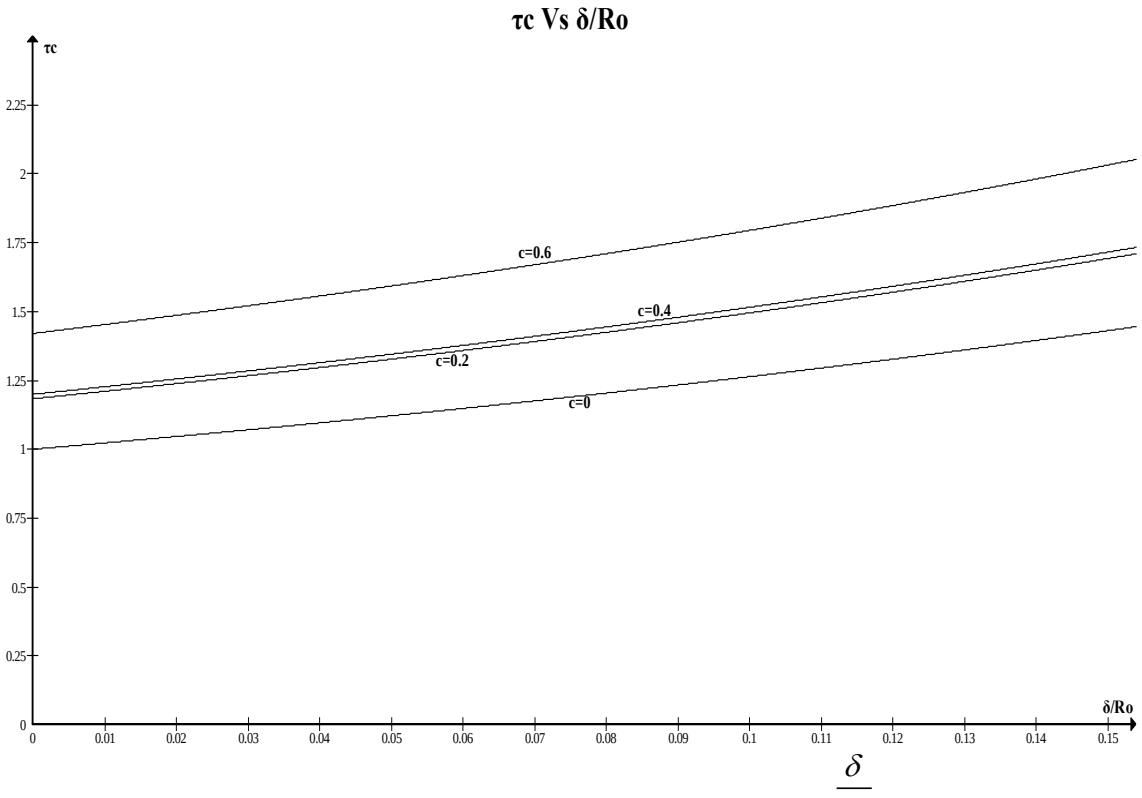

Figure 7 Variation of shear stress at critical stenosis height, τ_c with $\frac{\delta}{R_0}$ for different C

The trend of the variation of shear stress at the critical stenosis height τ_c is similar to that of the shear stress at stenosis throat τ_s. It is further observed that τ_c assumes significantly lower magnitude in comparison to τ_s for the same values of $\frac{\delta}{R_0}$ and C (**Figure 8**).

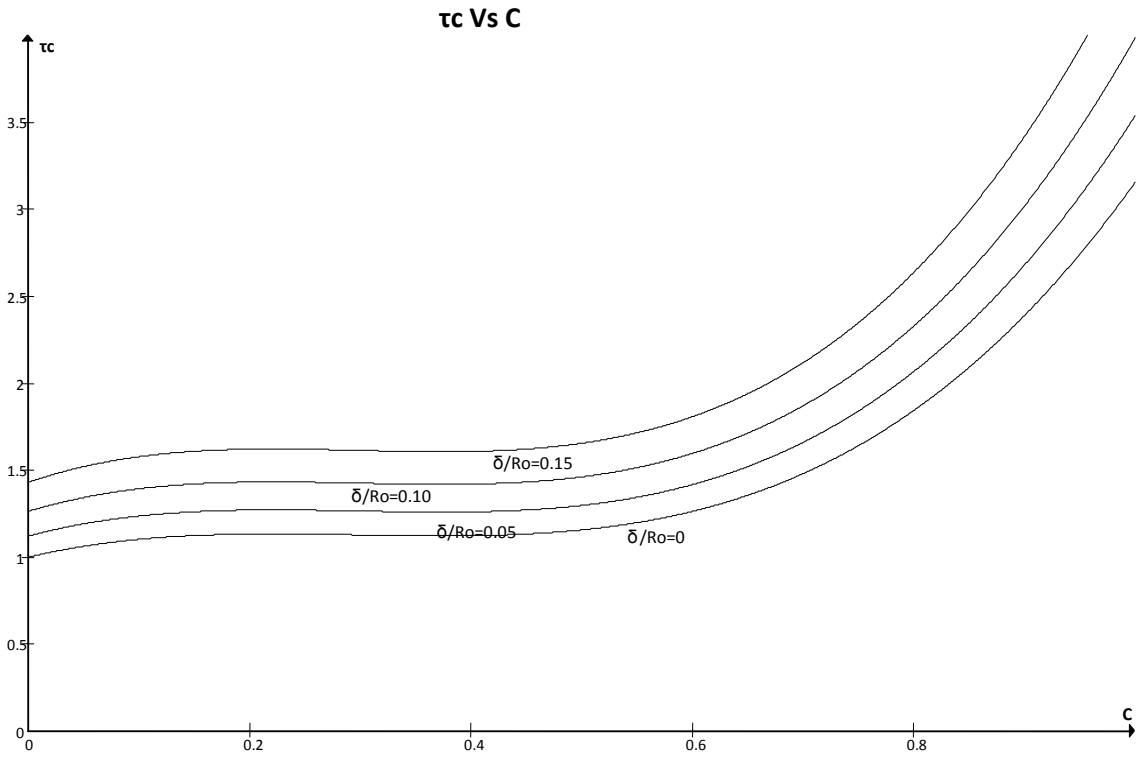

Figure 8 Variation of shear stress at critical stenosis height, τ_c with C for different $\frac{\delta}{R_0}$

CONCLUSION

This theoretical analysis provides a scope in bringing out many interesting results on rheological properties of blood flow through narrow multiple stenosed arteries treating two phase model of blood. This study provides a scope for estimating the role of various parameters involved in the analysis and to ascertain which of the parameters have the most dominating role in formation and development of this arterial disease. The investigation bears the potential to examine the severity of multiple stenoses. Under the purview of a single study with two phase model of blood it is expected that the results will help the physicians in predicting the stenotic range, the critical location and the severity of the disease so that they may take crucial decision for treatment through medicine or through surgery. Further careful investigations are suggested to address the problem more realistically and to overcome the restrictions imposed on the present work.

REFERENCES

1. Ahmed, P.S. and Giddens., D.P. Velocity measurements in steady flow through axisymmetric stenosis at moderate Reynolds number,J. Biomech, 16: 505-516, (1983)
2. Caro, C.G., Pedley, T.J., Schroter, R.C. and Seed, W.A. The Mechanics of the circulation, Oxford Medical, N.Y., 1978.
3. Chakravarty, S. and Mandal, P.K., Mathematical Modelling of blood flow through an overlapping stenosis, Math. Comput. Model, 19: 59-73, (1994)
4. Charm, S.E. and Kurland, G.S., Blood Flow and Micro Circulation. John Wiley, Newyork, 1974.
5. Charm, S.E. and Kurland, G.S. Blood Rheology in Cardiovascular Fluid Dynamics, Academic Press, London, 1965.
6. Cokelet, G.R. The Rheology of Human Blood: In Biomechanics, Prentice-Hall, Englewood Cliffs, N.J., 1972.
7. Haynes, R.H., Physical Basis on dependence of blood viscosity on tube radius, Am. J. Physiol,198:1193-1205, (1960)
8. Ku, D.N., Blood flow in arteries, Ann. Rev. Fluid Mech, 29:399-434, (1997)
9. Sakalak, R., Mechanics of Microcirculation: In Biomechanics, Its Foundation and Objectives, Prentice Hall Publ. Co. Englewood Cliffs, 1972.

10. Srivastava, L. M., Edemeka, U. E. and Srivastava, V. P., Effects of external body accelerations on blood flow, Jpn. J. Appl. Phys, 33: 3648-3655, (1994)
11. Srivastava, L.M. and Srivastava, V.P., On Two-Phase Model of Pulsatile Blood Flow with Entrance Effects, Biorheol, 20: 761-777, (1983)
12. Srivastava, V.P., A Theoretical model for blood flow in small vessels, Applc. Appl. Maths, 2:51-65, (2007)
13. Srivastava, V.P. and Rastogi, Rati., Effects of hematocrit on impedance and shear stress during stenosed artery catheterization, Applc. Appl. Math, 4: 98-113, (2009)
14. Srivastava, V.P. and Srivastava, Rashmi., Particulate suspension blood flow through a narrow catheterized artery, Comput. Math. Applic, 58:227-234, (2009)
15. Srivastava, V.P., Arterial blood flow through a non-symmetrical stenosis with applications, Jpn. J. Appl. Phys, 34: 6539-6545, (1995)
16. Young, D.F. and Tsai, F.Y., Flow characteristics in models of arterial stenosis-I. Steady Flow, J. Biomech,6: 395-410, (1973)
17. Young, D.F., Effects of a time-dependent stenosis of flow through a tube, J. Eng. Ind, 90: 248-254, (1968)
18. Young, D.F., Fluid mechanics of arterial stenosis, J. Biomech. Eng, 101: 157-175, (1979)
19. Medhavi, A., On macroscopic two-phase arterial blood flow through an overlapping stenosis,E-Journal of Science & Technology, 6(5):19-31, (2010)
20. Basu Mallik, B. and Nanda, S.P., A Mathematical Analysis Of Blood Flow Through Stenosed Arteries: A Non-Newtonian Model, IEM International Journal of Management & Technology(IEMIJMT), Vol-1(2): 41-45,(2012)
21. Nanda, S.P. and Bose, R.K., Mathematical Analysis on Blood Flow through a Flexible Stenosed Artery, IJCME, 2(1): 17-30,(2012)
22. Nanda, S.P. and Bose, R.K., Blood Flow Through A Flexible Artery In Presence Of Stenosis- A Mathematical Study, JMCMS, 6(2): 859-874,(2012)

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